Lesson 9-1: The Tangent Ratio

Trigonometry

For thousand of years astronomers, navigators and surveyors have had to figure out position and distance using ratios of known measurements. We worked with this last chapter with problems in which we knew the measures of two of the sides. We set up similar triangles and used the ratios of the two sets of corresponding sides to determine the third.

But what if we only know the measure of one of the sides and don't have similar triangles? How can we work with that? The answer is found in trigonometry.

An investigation of ratios

If you haven't done so yet, turn to page 470 in your text and work through the investigation exercise. I'll wait here for you…

What was your conjecture about the ratios of the sides? If you were careful with your measurements, you found that the ratio was the same no matter the size of the triangle. This is what is called the tangent ratio and is the first of the ***trigonometric ratios***.

The Tangent Ratio

The tangent ratio for a given angle of a right triangle is the ratio of the opposite side to the adjacent side. We will say the **tangent of** $\angle A$ and abbreviate it as **tan *A***:

$$\tan A = \frac{opposite}{adjacent}$$

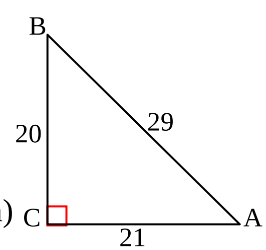
If you turn to page 731 of the text you will find a table of trigonometric ratios. This is what we used before we had these nice TI calculators. This table lists the tangents for all integral angles from 0 to 90. If you wanted to determine the tan 10.5° for instance, you'd have to interpolate between the value for tan 10 and tan 11. What a pain!

TI calculators to the rescue!

To find the tangent of an angle on your TI, simply press the *TAN* button, then enter the measure of the angle and hit *ENTER*. Try it a few times integral numbers and check the answer against trig table. You can see that the values in the trig table are rounded.

One thing, make sure your calculator is set to *degree mode*. It likely is, but you will get weird (wrong) answers is it isn't. To double check, hit the *MODE* key. The third line says *Radian Degree*. Make sure the *Degree* word is highlighted (use the arrow keys). Let's not worry about radians for now. ☺ You'll hit that next year.

A simple example



Write the tangent ratios for $\angle A and \angle B$.

$$\tan A = \frac{opp}{adj} = \frac{20}{21} \text{ and } \tan B = \frac{opp}{adj} = \frac{21}{20} \text{ (leave in fraction form)}$$

Great, so how do we <u>use</u> this?

Back to the question at the beginning…what if we don't have similar triangles from which we can set up a ratio statement? Consider the following example.

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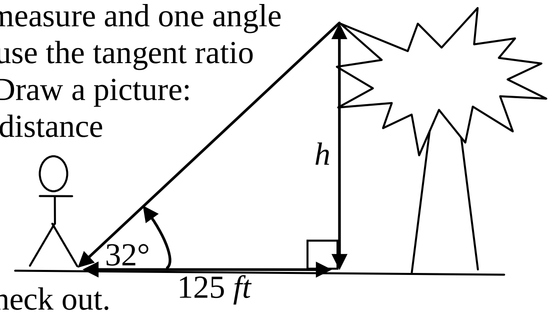
To measure the height of a tree, Alma walked 125 ft from the tree and measured a 32° angle from the ground to the top of the tree. Estimate the height of the tree.

What information do we have? We have one distance measure and one angle measure. We're missing the 2nd distance measure. To use the tangent ratio we need to determine the opposite and adjacent sides. Draw a picture: you can now see the height is the opposite side and the distance is the adjacent. Now write and solve the equation:

$$\tan 32 = \frac{h}{125} \text{ or } h = 125 \cdot \tan 32 = 78.108 \approx 78\,ft$$



There is another example in the book that you should check out.

Going backwards…the inverse tangent

What if you knew the lengths of the legs and wanted to determine the angle? In other words you would be asking "what is the angle whose tangent is *x*?" This is called the ***inverse tangent*** and is written $\tan^{-1}$. For instance "the angle *A* whose tangent is 0.75" is written: $m\angle A = \tan^{-1}(0.75)$. You need to be careful here… $\tan^{-1}(0.75) \neq \tan(0.75)^{-1}$!

The first is the inverse tangent (and is 36.87), the second is $\frac{1}{\tan(0.75)}$ (and is 76.39).

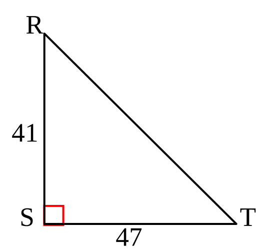
The $\tan^{-1}$ on your calculator is the 2nd function of the *TAN* key. So to find the $\tan^{-1}(0.75)$ hit *2nd*, then *TAN*, then 0.75, then *ENTER*.

Example

Find $m\angle R$ to the nearest degree.

Opposite = 47
Adjacent = 41

$$m\angle R = \tan^{-1}\frac{47}{41} = 48.90.$$



A definition

One very good application of this is to surveying. Remember the last time you went over a mountain pass? It is very likely that you saw a sign on the downhill side that said something this:

The 10% is the ***grade*** of the road and indicates how steep it is. It is basically the ***slope*** of the road and just like slope in the equation of a line is $\frac{rise}{run}$. So, how do you get a slope from 10%? Well think about it: what does percent mean? It is some part of 100. So 10% is a fraction: $\frac{10}{100}$. Again, as a grade, this is rise over run. The rise (or change in *y*) is 10 and the run (change in *x*) is 100. You can use this to determine what angle the road is to the horizontal: using this 10% grade example: $\tan^{-1}(\frac{10}{100}) = 5.7016 \approx 5.7°$

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Wrap up

Here are two questions, that if you take the time to think through, will help you understand the tangent ratio better:

1. Without using a calculator, how would you find the angle whose tangent equals 1?
2. Without using a calculator, how would you find tan 60°?

As a hint, think back to our special triangles from lesson 7-3. I'm not going to give you the answers here…check with me in class if you need help. ☺

Homework Assignment

p. 472 #1-22, 26-29, 31-43 odd, 53-54